

## SCOUR DURATION AT STRUCTURES AT STEADY FLOW CONDITIONS

**Oskars LAUVA**, Water Engineering and Technology Department, Riga Technical University, Azenes 16/20, LV-1048, Riga, Latvia, [lauva.oskars@gmail.com](mailto:lauva.oskars@gmail.com) (corresponding author)

**Boriss GJUNSBURGS**, Water Engineering and Technology Department, Riga Technical University, Azenes 16/20, LV-1048, Riga, Latvia, [gjunsburgs@bf.rtu.lv](mailto:gjunsburgs@bf.rtu.lv)

One of the reasons of the bridge crossings failure or damage in flow is the unpredicted depth of scour near foundations. The aim of this study is to find the equilibrium time near elliptical guide banks. Analysis of the literature shows that there are no methods or formulas to calculate equilibrium time of scour near elliptical guide banks. In the formulas used in calculating equilibrium time at piers or abutments, different parameters are not taken into consideration. These parameters include: contraction rate of the flow, Froude number, bed layering, sediment movement parameters, local flow modification, ratio relative local and critical velocities, and relative depth. The differential equation of the bed sediment movement in clear water was used and method for computing equilibrium time of scour near elliptical guide banks was elaborated. New hydraulic threshold criterion is proposed for the calculation of equilibrium time of scour. Computer modeling results were compared with equilibrium time of scour which were calculated by the presented method and they were in agreement.

*Keywords: contraction rate of the flow, equilibrium time, local scour.*

### INTRODUCTION

Different authors in order for level of the bridge foundations prediction use formulae where equilibrium time of scour is as one of the parameters. The equilibrium depth of scour development under steady flow can be reached in equilibrium time. Incorrect prediction of the depth of scour and consequently the level of the foundation for abutments, piers, guide banks or spur dikes may lead to severe damages of bridge structures and cause considerable economic and financial losses.

The equilibrium time of scour at bridge piers, abutments and spur dikes were studied, among others, Melville and Chiew (1999), Ballio and Orsi (2001), Lauchlan et al. (2001), Coleman et al. (2003), Gjunsburgs and Neilands (2004), Dey and Barbhuiya (2005), Grimaldi et al. (2006), Cardoso and Fael (2010), Gjunsburgs et al. (2010, 2014), Ghani et al. (2011), Mohammadpour et al. (2011), Abou-Seida et al. (2012).

Scour evaluation at clear water conditions never cease completely, so threshold criteria has to be found when scour development in time has reduced to a negligible value. The various threshold criteria proposed in literature usually are assumed that equilibrium has been reached when the depth of scour evaluation is less than 5 % of the pier diameter within a period of 24 hours (Melville and Chiew, 1999) or less than 5 % of the flow depth or abutments length (Coleman et al., 2003), or again less than 5 % of the 1/3 of the pier diameter (Grimaldi et al., 2006). All these criteria for equilibrium time of scour reference only on the geometrical size of the bridge structures.

Analysis of the literature shows that today there are no methods or formulas to predict equilibrium time of scour near elliptical guide banks at clear water conditions, while in the available formulas for calculating equilibrium time of scour at piers and abutments some important parameters of the flow and river bed are not taken into consideration.

The aim of this paper is to find the equilibrium time near elliptical guide banks in clear water conditions by using the differential equation of the bed sediment movement in clear water. Solution of that equation, which describes scour development in time, allows easily finding equilibrium time.

New hydraulic criterion is proposed to determine the equilibrium time of scour. It is found that equilibrium time of scour is depending on the following parameters: contraction rate of the flow, Froude number, bed layering, grain size diameter, local flow velocity near structure, ratio of local velocity to critical one, and are changing with relative depth of scour.

The equilibrium time of scour in tests is calculated by using grain size diameter  $d_{50}$ , which is found from uniform sand granulometric curve. The test results of scour evaluation in time with duration of 7 hours were prolonged till the equilibrium stage of scour. Computer modeling of equilibrium depth and time of scour by early proposed method by

Gjunsburgs et al. (2006) was made and compared with equilibrium time of scour at the elliptical guide bank calculated by the presented method.

## EXPERIMENTAL SETUP

The tests were carried out in a flume 3.5 m wide and 21 m long (Figure 1). The tests were carried out under open flow conditions, while studying the flow distribution between the channel and the floodplain. Experimental data for the open-flow conditions are presented in Table 1.

The fixed bed tests were performed for different flow contractions and Froude numbers in order to investigate the local velocity and the water level changes in the vicinity of the guide banks and along it.

The aim of the sand bed tests was to study the scour process, the changes in the local velocity, the effect of different hydraulic parameters, the flow contraction rate, the grain size, stratification of the model bed and the scour development in time.

The openings of the bridge model were 50, 80, 120, and 200 cm (see Figure 1). The flow contraction rate  $Q/Q_b$  (where  $Q$  is the flow discharge and  $Q_b$  is the discharge in the bridge opening under open-flow conditions) varied respectively from 1.56 to 5.69. The depth of water on floodplain was 7 cm and 13 cm, while the Froude numbers varied from 0.078 to 0.134.

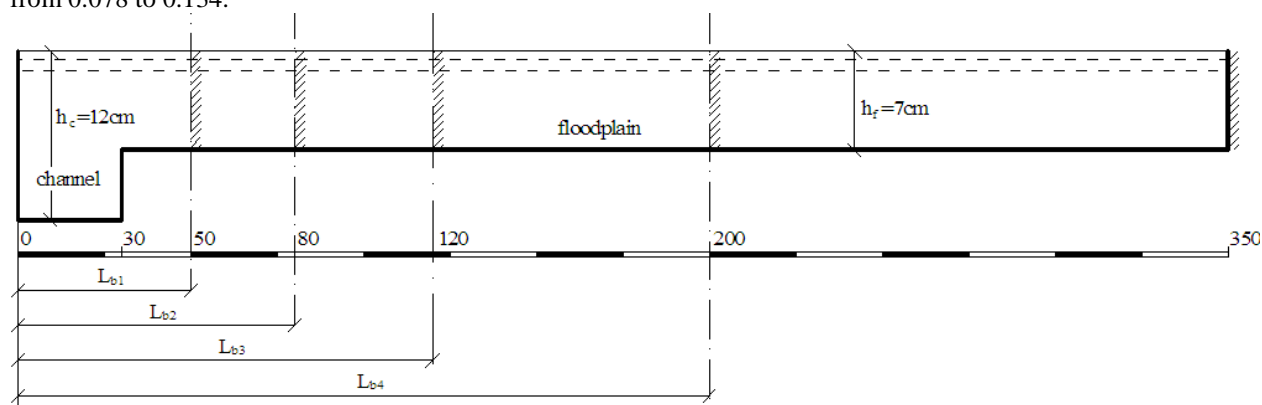


Figure 1. Cross-section view of the experiment flume.

The tests were carried out under clear-water conditions. The sand was placed 1 m up and down the contraction of the flume. The mean grain size was 0.24 mm and 0.67 mm.

Table 1. Test data for open flow conditions

Test	L [cm]	h [cm]	V [cm/s]	Q [l/s]	Fr	Re <sub>c</sub>	Re <sub>f</sub>
L1	350	7	6.47	16.60	0.078	7500	4390
L2	350	7	8.58	22.70	0.103	10010	6060
L3	350	7	10.3	23.60	0.124	12280	7190
L7	350	13	7.51	35.48	0.066	13700	9740
L8	350	13	8.74	41.38	0.075	16010	11395

The tests were carried out with one floodplain model and one side contraction of the flow. The dimension of the upper part of an elliptical guide bank, particularly the length was calculated according to the Latishenkov (1960) method and was found to be dependent on the flow contraction rate and the main channel width. The length of the lower part of the guide bank was assumed to be half of the upper part.

## METHOD

The differential equation of equilibrium for the bed sediment movement in clear-water conditions was used to create the calculation method for equilibrium time of scour at elliptical guide banks:

$$\frac{dW}{dt} = Q_s, \quad (1)$$

where:  $W$  = the volume of the scour hole at elliptical guide bank, which, according to the test results, is equal to  $1/5\pi m^2 h_s^3$ ;  $t$  = time; and  $Q_s$  = the sediment discharge out of the scour hole.

The calculation method is described in more details and can be found in Gjunsburgs and Lauva (2015). Equilibrium time of scour near elliptical guide banks reads:

$$t_{equil} = 4D_{equil} \cdot h_f^2 (N_{equil} - N_{i-1}), \quad (2)$$

where:  $t_{equil}$  = equilibrium time of scour;  $h_f$  = flow depth in the floodplain;  $D_{equil}$  = calculated parameter, depending on the steepness of the scour hole, local flow velocity, critical velocity, and grain size of the bed material;  $N_{equil}$ , and  $N_{i-1}$  = calculated parameters, depending from relative depth of scour.

The sequence to calculate the equilibrium time of scour is the next, the equilibrium depth of scour at elliptical guide banks is found (Gjunsburgs et al., 2006a):

$$h_{equil} = 2h_f \left[ \left( \frac{V_{lel}}{\beta V_0} \right)^{0.8} - 1 \right] \cdot k_\alpha \cdot k_m, \quad (3)$$

where:  $h_{equil}$  = equilibrium depth of scour;  $h_f$  = flow depth in the floodplain;  $V_{lel}$  = local flow velocity at the elliptical guide bank;  $\beta$  = coefficient of reduction in the sediment critical velocity due to vortex structures;  $V_0 = 3.6d_i^{0.25}h_f^{0.25}$  = the critical velocity at the plain bed;  $d_i$  = grain size of the bed materials;  $k_\alpha$  = a coefficient depending on the flow crossing angle; and  $k_m$  = a coefficient depending on the side-wall slope of guide banks.

When local velocity  $V_{lel}$  becomes equal to critical velocity  $\beta V_0$ ,  $t_{equil} = \infty$ . Criteria to evaluate threshold is needed to appoint to calculate equilibrium time of scour.

## ANALYSIS OF THE METHOD

To analyze the method, Equation (2) is transformed to a form that shows clearly that it contains dimensionless parameters and characteristics of the flow and riverbed:

$$t = \frac{(N_i - N_{i-1})(\pi m h_f^2) \left( \frac{d}{h_f} \right)^{0.25}}{\left\{ \frac{P_k}{2} \left[ \left( \frac{Q}{Q_b} \right)^2 - 1 \right] + \frac{1}{2} P_{kb} \sqrt{\frac{1}{Fr}} \left[ \left( \frac{Q}{Q_b} \right)^2 + 1 \right] + P_{kb} \right\}^2 \cdot 2A_1 \varphi^4 g^2 h^2}, \quad (4)$$

where:  $N_i$  and  $N_{i-1}$  = calculated parameters, depending on relative depth of scour;  $m$  = steepness of the scour hole;  $h_f$  = water depth in the floodplain;  $d$  = grain size of the bed materials;  $Q/Q_b$  = flow contraction rate;  $P_K = V_K^2/gh$  = kinetic parameter of flow in contraction in open-flow conditions;  $P_{kb} = V^2/gh_f$  = kinetic parameter of the open flow in natural conditions;  $Fr/i$  = ratio of the Froude number to the river slope;  $h/h_f$  = relative depth of the flow;  $g$  = acceleration due to gravity;  $h$  = average depth of the flow in the contracted section;  $A_1$  = calculated parameter, depending from local velocity, critical velocity and grain size of the bed material; and  $\square$  = shear stress.

In the general form, the equilibrium time of scour is a function of the following parameters:

$$t = f \left( \frac{Q}{Q_b}; P_k; P_{kb}; \frac{Fr}{i}; \frac{d}{h_f}; \frac{\beta V_0}{V_{lel}}; \frac{h}{h_f}; \frac{h_s}{h_f}; N_{i-1} \right), \quad (5)$$

where:  $Q/Q_b$  = flow contraction rate;  $P_K$  = kinetic parameter of flow in contraction in open-flow conditions;  $P_{kb}$  = kinetic parameter of the open flow;  $Fr/i$  = ratio of the Froude number to the river slope;  $d/h_f$  = dimensionless grain size;  $\square \square V_0/V_{lel}$  = ratio of the sediment critical velocity at which the sediment movement starts to the local velocity;  $\square$  = coefficient of reduction in the sediment critical velocity due to vortex structures;  $h/h_f$  = relative flow depth;  $h_s/h_f$  = relative scour depth; and  $N_{i-1}$  = calculated parameter, depending from relative depth of the flow.

## RESULTS

At the head of the elliptical guide bank, we observe the concentration of streamlines, a sharp drop in water level, and a local increase in the velocity. Locally modified flow near the guide banks is forming the scour hole. Figure 3 illustrates the scour depth  $h_s$ , respective variations in the local  $V_{lel}$  and the critical  $\beta V_{0t}$  velocities, as measured experimentally and calculated in steady flow including the layer with uniform sand. With the scour depth increase, the local velocity is reducing and the critical one is increasing (Figure 2).

The ratio of critical velocity to the local one at the head of elliptical guide bank is accepted as threshold criteria in equilibrium time of scour calculation. According to computer modeling results the scour stops when local velocity  $V_{lel}$  becomes equal to critical velocity  $\beta V_{0t}$  or ratio of those velocities becomes equal to 1, and equilibrium is equal to infinity. The threshold criterion was checked and accepted equal for calculation equilibrium time of scour:

$$\frac{\beta V_{0t}}{V_{lel}} = \frac{\beta V_0}{V_{lel}} \left( 1 + \frac{h_{equil}}{2h_f} \right)^{1.25} = 0.985222 \cdot \quad (6)$$

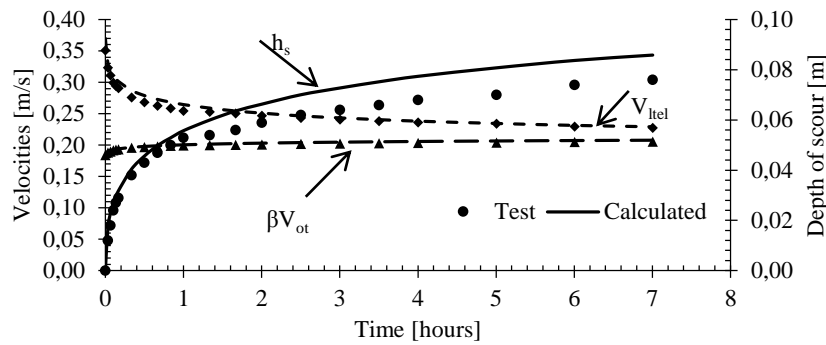


Figure 2. Changes in scour depth and in the local and critical velocities  $V_{tel}$  and  $\beta V_{ot}$  varying with time under steady flow with one-sand layer  $d_{50}=0.24$  mm; test EL 6.

Analysis of the method presented and test results confirmed the influence of contraction rate of the flow, Froude number, bed grain size diameter, relative local and critical velocities ratio, as well as relative depth of scour on equilibrium time of scour.

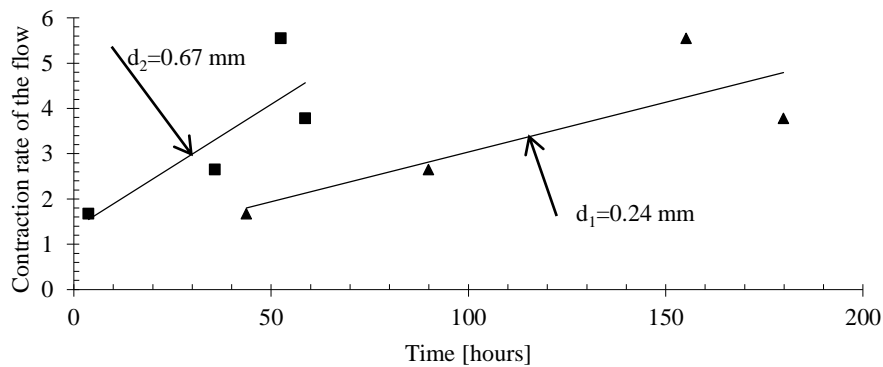


Figure 3. Contraction rate of the flow influence on equilibrium time of scour. Tests EL3, EL6, EL9, and EL12 with two different sand grain sizes  $d_1=0.24$  mm and  $d_2=0.67$  mm.

Flow contraction creates a series of events that also has an impact on the equilibrium time of scour, thereby if the flow contraction rate  $Q/Q_b$  increases, that leads to an increased equilibrium time of scour, consequently the greater the contraction rate of the flow  $Q/Q_b$  value is, the greater the equilibrium time of scour value becomes. Since finer sand particles are more easily scoured away, it takes greater time to achieve equilibrium stage in the case, when the sand is fine ( $d_1=0.24$  mm), than in the case with coarse ( $d_2=0.67$  mm) sand. The ratio of the critical velocity to local one  $\beta V_{ot}/V_{tel}$  is depending on contraction rate of the flow  $Q/Q_b$  therefore with increase of contraction rate of the flow the equilibrium time of scour is increasing (Figure 3).

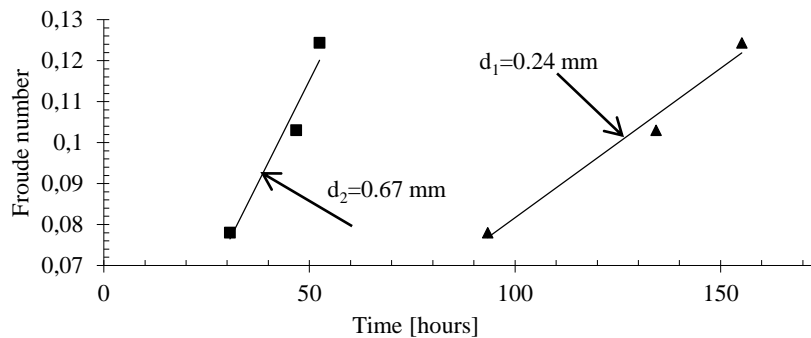


Figure 4. Froude number impact on equilibrium time of scour. Tests EL1, EL2 and EL3 with two different sand grain sizes  $d_1=0.24$  mm and  $d_2=0.67$  mm.

With an increase in the Froude number  $Fr$ , there is also an increase in the equilibrium time of scour, the greater the Froude number  $Fr$  value becomes, the further the scouring process continues in the scour hole, resulting also in a greater equilibrium time of scour. With finer sand ( $d_1=0.24$  mm) the equilibrium time is greater, the scouring process takes longer to achieve equilibrium stage; with coarser sand ( $d_2=0.67$  mm) on the other hand the scouring process ends more quickly, resulting in a lesser equilibrium time of scour value (Figure 4). The greater the grain size diameter of the river bed is, the less the equilibrium time of scour becomes.

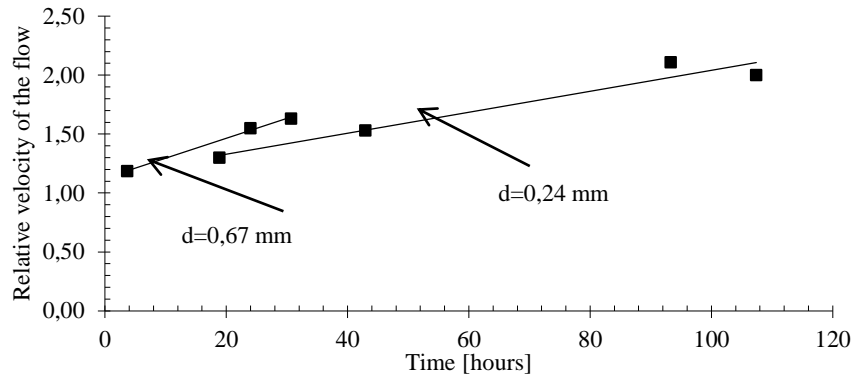


Figure 5. Relative velocity of the flow influence on the equilibrium time of scour. Tests EL1, EL4, EL7 and EL10 with two different sand grain sizes  $d_1=0.24$  mm and  $d_2=0.67$  mm.

Flow velocity  $V$  is one of the fundamental scouring agents; when the local flow velocity  $V_{tel}$  at the elliptical guide bank exceeds the critical value of sediments, the scouring process begins. Since coarser sand particles ( $d_2=0.67$  mm) are heavier, it is more difficult to scour them away, so with an increase of the relative velocity of the flow  $V_{tel}/V_0$  the increase in the equilibrium time of scour is medium, however for the finer sand ( $d_1=0.24$  mm) the increase in equilibrium time is more accelerating with the increase of the relative velocity of the flow  $V_{tel}/V_0$ . So the greater the local flow velocity  $V_{tel}$  is and, at the same time, the smaller the critical flow velocity  $V_0$  for the finer the sand ( $d_1=0.24$  mm, instead of  $d_2=0.67$  mm) is, the greater the equilibrium time of scour will become (Figure 5).

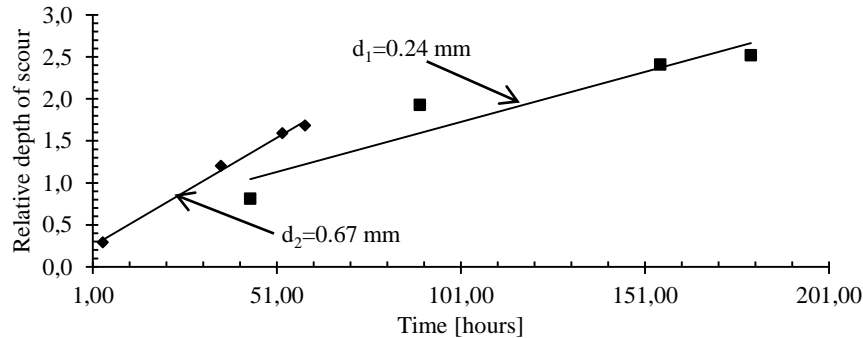


Figure 6. Equilibrium time versus relative equilibrium depth of scour with sand particle sizes  $d_1=0.24$  mm and  $d_2=0.67$  mm.

When the scouring process continues in the scour hole, it takes longer time to achieve time to equilibrium scour. In the case of coarser sand ( $d_2=0.67$  mm) the depth of scour is achieved faster. Thus resulting in lesser relative depth of scour and at the same time lesser time to equilibrium scour, however with finer sand ( $d_1=0.24$  mm) on the contrary, it takes more time to achieve equilibrium scour depth, since also the scour depth increases, consequently increasing the relative depth of scour  $h_{equil}/h_f$ . So relative depth of scour  $h_{equil}/h_f$  is connected with equilibrium time of scour in a direct way – if the relative depth of scour  $h_{equil}/h_f$  becomes greater, the equilibrium time of scour becomes greater as well (Figure 6).

Using threshold criteria (Eq. 6), equilibrium depth of scour  $h_{equil}$  (Eq. 3),  $A$ ,  $D$ ,  $N$  and finally the equilibrium time  $t_{equil}$  (Eq. 2) is calculated.

Comparison of some results of equilibrium time of scour calculated by method presented and found by computer modeling for  $d_{50}=0.24$  mm is presented in Table 2.

Table 2. Comparison equilibrium time of scour calculated by computer modeling and by proposed methods for  $d_{50}=0.24$  mm

TEST	$Q/Q_b$	D	$N_i-N_{i-1}$	$t_{comp.}$ [hours]	$t_{form.}$ [hours]	$t_c/t_f$	$bV_{ot}/V_{itel}$	Fr
EL1	5.27	104.71	1.89	96.0	93.33	1.03	0.985	0.078
EL4	3.66	166.54	1.37	92.1	107.42	0.86	0.985	0.078
EL7	2.60	450.47	0.20	45.0	42.96	1.05	0.985	0.078
EL10	1.56	957.28	0.04	18.0	18.89	0.95	0.985	0.078
EL2	5.69	52.28	5.46	132.0	134.25	0.98	0.985	0.103
EL5	3.87	47.49	4.09	100.8	91.36	1.10	0.985	0.103
EL8	2.69	130.76	1.55	90.0	95.57	0.94	0.985	0.103
EL11	1.66	619.50	0.10	30.5	29.96	1.02	0.985	0.103
EL3	5.55	40.54	8.14	153.6	155.18	0.99	0.985	0.124

TEST	Q/Q <sub>b</sub>	D	N <sub>i</sub> -N <sub>i-1</sub>	t <sub>comp.</sub> [hours]	t <sub>form.</sub> [hours]	t <sub>e</sub> /t <sub>f</sub>	bV <sub>ot</sub> /V <sub>itel</sub>	Fr
EL6	3.78	39.32	9.72	151.2	179.83	0.84	0.985	0.124
EL9	2.65	55.08	3.47	84.0	89.87	0.93	0.985	0.124
EL12	1.67	467.54	0.20	45.0	43.76	1.03	0.985	0.124

Computer modeling of scour evaluation in time near elliptical guide banks at clear water conditions was used (Gjunsburgs et al., 2006a) to prolong test results to equilibrium depth and time of scour.

Comparison of equilibrium times of scour calculated by computer modeling and by Eq. (2) has been made; results are in good agreement (Table 2).

## CONCLUSIONS

The flow pattern at the head of the elliptical guide banks is modified, the concentration of streamlines, a sharp drop in water level, local increase in the velocity, circulation and scour hole was observed. Locally modified flow near the head of the guide banks is forming the scour hole.

The equilibrium depth of scour development under steady flow can be reached in equilibrium time. An analysis of the literature shows that there are no methods or formulas to calculate equilibrium time of scour near elliptical guide banks.

The differential equation of the bed sediment movement in clear water was used and the method for computing equilibrium time of scour near elliptical guide banks was considerably elaborated. The test results in flume with duration of 7 hours were prolonged till the equilibrium stage by calculation of scour evaluation in time (Gjunsburgs et al., 2006b).

It is confirmed by method elaborated that equilibrium time of scour is depending on contraction rate of the flow, Froude number, grain size diameter, local flow velocity near structure, and ratio of local velocity to critical one, and is changing with different relative depth of scour. Dependence of equilibrium time of scour from some parameters is presented in Figures 3–6.

With the scour depth increase, the local velocity is reducing and the critical one is increasing. According to Gjunsburgs et al. (2006a), the scour evaluation stops when local velocity  $V_{itel}$  becomes equal to critical velocity  $\beta V_{ot}$  and the ratio of those velocities becomes equal to 1. In that case equilibrium time goes to infinity  $t_{equil} = \infty$ . As scour evaluation at clear water conditions never cease completely, the threshold criterion is needed to accept when scour development in time has reduced to a negligible value. The new criterion as  $\beta V_{ot}/V_{itel} = 0.985222$  is checked and accepted for equilibrium time of scour calculation.

The equilibrium time of scour in tests is calculated by using grain size diameter  $d_{50}$ , which is found from uniform sand granulometric curve. The equilibrium time for other grain size diameters, for example,  $d_{10}$ ,  $d_{16}$ ,  $d_{84}$ ,  $d_{90}$  of the same uniform sand will be very different.

Using the new threshold criteria and values  $h_{equil}$ ,  $A$ ,  $D$ ,  $N$  and finally  $t_{equil}$  can be calculated using Eq. (2).

Computer modeling results were compared with equilibrium time of scour for grain size  $d_{50}$  calculated by the method presented (Eq. 2) and they were in agreement (Table 2).

## REFERENCES

1. Abou-Seida, M. M.; Elsaheed, G. H.; Mostafa, T. M.; Elzahry, E. F. 2012. Local scour at bridge abutments in cohesive soil. Journal of Hydraulic Research Vol. 50, No. 2, pp. 171–180. <http://dx.doi.org/10.1080/00221686.2012.654668>
2. Ballio, F., Orsi, E. 2001. Time evolution of scour around bridge abutments. Water Engineering Resources, Vol. 2, No. 4, pp. 243–259.
3. Cardoso, A. H., Fael, C. M. S. 2010. Time to equilibrium scour at vertical wall bridge abutments. Proceedings of the ICE-Water Management, Vol. 163, Iss. 10, 509–513. <http://dx.doi.org/10.1680/wama.900038>
4. Coleman, S. E., Lauchlan, C. S., Melville, B. W. 2003. Clear water scour development at bridge abutments. Journal of Hydraulic Research, Vol., 42, Iss. 5, pp. 521–531. <http://dx.doi.org/10.1080/00221680309499997>
5. Dey, S., Barbhuiya, A.K. 2005. Time variation of scour at abutments. Journal of Hydraulic Engineering, Vol. 131, Iss. 1, pp. 11–23. [http://dx.doi.org/10.1061/\(ASCE\)0733-9429\(2005\)131:1\(11\)](http://dx.doi.org/10.1061/(ASCE)0733-9429(2005)131:1(11))
6. Ghani, A., Azamathullah, H., Mohammadpour, R. 2011. Estimating time to equilibrium scour at long abutment by using genetic programming. 3rd International Conference on Managing Rivers in the 21st Century: Sustainable Solutions for Global Crisis of Flooding, Pollution and Water Scarcity, 369–374, Penang, Malaysia.
7. Gjunsburgs, B., Neilands, R. 2004. Local velocity at bridge abutments on plain rivers. Proc. River Flow 2004, in Greco, Carravetta & Della Morte (eds), (1), 443–448, Napoly, Italy. <http://dx.doi.org/10.1201/b16998-58>
8. Gjunsburgs, B., Govsha, E., Neilands, R. 2006. Local Scour at the Elliptical Guide Banks. ICSE-3, 3rd International Conference on Scour and Erosion, Netherland, Amsterdam, pp. 120–128.
9. Gjunsburgs, B., Govsha, E., Neilands R. 2007. Scour development at elliptical guide banks during multiple floods. Harmonizing the demands of art and nature in hydraulics, 32nd Congress of IAHR (CD), pp. 1–10, Italy.
10. Gjunsburgs, B., Jaudzems, G., Govsha, E. 2010. Scour at elliptical guide banks under stratified bed conditions: equilibrium stage. Proceeding of International Scientific Conference of People, Buildings and Environment, Krtiny, pp. 24–30.

11. Gjunzburgs, B.; Lauva, O.; Neilands, R. 2014. Equilibrium time of scour near structures in plain rivers. The 9th International Conference "Environmental Engineering", Vilnius, Lithuania. <http://dx.doi.org/10.3846/enviro.2014.077>
12. Gjunzburgs, B.; Lauva, O. 2015. Time of scour at elliptical guide banks. 2nd International Workshop on Hydraulic Structures: Data Validation, Coimbra, Portugal, 8–9 May 2015.
13. Grimaldi, C., Gaudio, R., Cardoso, A. H., Calomino F. 2006. Local scouring at bridge piers and abutments: time evolution and equilibrium. Proceeding River Flow 2006, Ferreira, Alves, Leal & Cardoso (eds), (1), pp. 1657–1664, Lisbon, Portugal.
14. Latishenkov, A. M. 1960. Questions of artificially contracted flow. Moscow: Gostroizdat (in Russian).
15. Lauchlan, C. S., Coleman, S. E., Melville, B. W. 2001. Temporal scour development at bridge abutments. Proceedings of the XXIX Congress of the International Association of Hydraulics Research, pp. 738–745, Beijing.
16. Melville, B. W., Chiew, Y. M. 1999. Time scale for local scour at bridge piers. Journal of Hydraulic Engineering, Vol. 125, Iss. 1, pp. 59–65. [http://dx.doi.org/10.1061/\(ASCE\)0733-9429\(1999\)125:1\(59\)](http://dx.doi.org/10.1061/(ASCE)0733-9429(1999)125:1(59))
17. Mohammadpour, R., Ghani, A. A., Azamathullah, H. M. 2011. Estimating time to equilibrium scour at long abutment by using genetic programming. 3rd International Conference on Managing Rivers in the 21st Century: Sustainable Solutions for Global Crisis of Flooding, Pollution and Water Scarcity, pp. 369–374, Penang, Malaysia.