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EXTENSION OF WEIBULL MODEL FOR DESCRIBING OF DRIED APPLE REHYDRATION

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Sample of Ligol variety apples (slices of 3 and 10 mm thickness and cubes of 10 mm thickness) were dried using following methods: natural convection (drying air velocity amounted to 0.01 m/s), forced convection (0.5 and 2 m/s), fluidized bed drying (6 m/s). The drying air temperature was kept at 50, 60, and 70°C. The dried apples samples were rehydrated in distilled water at 20, 45, 70, and 95°C. The Weibull model given for describing time dependence of the moisture content change was fitted to experimental data and model parameters were determined by multiple regression analysis. The variation of model parameters with characteristic particle dimension (*L*), drying air velocity (*v*), drying air temperature (t_d), and rehydration temperature (t_r) described as multiplication type. By using these verification of parameters, extended Weibull model for describing combine effects of *L*, *v*, t_d, t_r, and drying time was derived and the parameters of the model were also determined by multiple regression analysis. The accuracies of both models were measured using the determination coefficient (R^2), mean bias error (MBE), root mean square error (RMSE), reduced chi-square (χ^2), and t-statistic method. The Weibull model (R^2=0.8319-0.9957, MBE=-0.0044-0.0110, RMSE=0.0189-0.1248, χ^2=0.0004-0.0180, and t-stat=0.0149-0.2875) and the extended Weibull model (R^2=0.9130-0.9948, MBE=-0.0209-0.0377, RMSE=0.0230-0.0719, χ^2=0.0007-0.0057, and t-stat=0.0389-1.2214) described the rehydration characteristics of dried apple satisfactorily. The extended model by taking into account the effect of *L*, *v*, t_d, and t_r on its parameters can be considered as more general one.

Keywords: apple, rehydration, Weibull model.

INTRODUCTION

The knowledge of the rehydration kinetics of dried products is important to optimise process from a quality point of view because rehydration is a key quality aspect for those dried products that have to be reconstituted before their consumption. Different transport mechanisms take place during rehydration of dried food products, namely molecular diffusion, convection, hydraulic flow, and capillary flow (Saguy et al. 2005). The mathematical modelling of the rehydration process is the most important aspect of rehydration technology. Different authors modelled discussed process. Some of them proposed theoretical models among others such based on Fick's law of diffusion. Górnicki (2011) used theoretical model to describing of dried apple and parsley root rehydration. Maldonado et al. (2010) and Markowski et al. (2009) used mentioned model for describing of dried mangoes and potato rehydration, respectively.

Empirical models such as the Peleg (1988) model, Weibull model (Garcia-Pascual et al. 2006) or Witrowa-Rajchert (1999) model are not derived from any physical laws but their application is relatively easy.

The Weibull model has been found to give satisfactory results in the description of rehydration of the dried materials. Ergűn et al. (2016) determined the rehydration kinetics of freeze dried (13.33 Pa absolute pressure, −48 °C condenser temperature) kiwi slices. Rehydration experiments were carried out in distilled water at a temperature of 18°C. The solid: liquid ratios were adjusted as 1:25, 1:50, 1:75, 1:100, and 1:125 (w:w). The kinetics of the moisture absorption of the kiwi slices was modelled by the application Weibull model. The results showed that the Weibull model was not well fitted to the experimental data due to low R^2 values (0.677–0.9232). Link et al. (2017) compared different drying methods (conductive multi-flash drying, air-drying, vacuum drying, and freeze-drying) with respect to rehydration kinetics of dried mangoes slices. Dried mangoes were immersed in distilled water (1:100 - dried mango weight: water weight) at 20°C and 80°C. The Exponential model, Peleg model and Weibull model were used for describing the rehydration of mango samples. However, the best statistical parameters were obtained fitting the Weibull model to the experimental data, thus this model was considered the most accurate for describing the rehydration kinetics of dried mangoes. Garcia-Pascual et al. (2005) examined of rehydration of air-dried *Morchella esculenta* mushrooms at 15, 20, 25, 30, 45, 55 and 70°C. Authors used Weibull model to describe the kinetics of rehydration. Rafiq et al. (2015) examined hydration behaviour with processing conditions of low amylose content parboiled rice which was dried at 40, 50 and 60°C. The rehydration characteristics of the parboiled rice was studied at various soaking temperatures (30, 40 and 50°C). Three semi-empirical models: Peleg model, Exponential model, and Weibull model were examined for fitting the experimental data of water uptake kinetics of parboiled paddy. The average R^2 value of fit was 0.963 for Peleg's equation and 0.973 for Exponential and 0.986 for Weibull equation. Machado et al. (1999) measured the moisture uptake by ready-to-eat corn breakfast cereal immersed in milk solutions, reconstituted from whole and skimmed milk powder under isothermal conditions at 5, 30 and 55°C. The Weibull model described the moisture uptake process by corn breakfast cereal adequately.

The objectives of this study were: (1) to apply the Weibull model to the description of moisture content changes during rehydration of dried apples, (2) to determine the effect of characteristic particle dimension, drying air velocity, drying air temperature, and rehydration temperature on model parameters. It turned out from the literature survey that there is no information on the subject undertaken in this study.

MATERIAL AND METHODS

High-quality Ligol apples bought at a Warsaw market were used in the research. The raw material was washed, peeled, and cut into slices of 3 and 10 mm thickness and cubes of 10 mm thickness. The apple samples were dried using following methods: natural convection (drying air velocity amounted to 0.01 m/s), forced convection (0.5 and 2 m/s), and fluidized bed drying (6 m/s). The drying air temperature was kept at 50, 60, and 70°C. The drying lasted until the constant weight of the dried material was attained. Dried material obtained in the same conditions was stored in air-tight glass container until it was used in the rehydration experiments.

The dried apples samples were rehydrated in distilled water at 20, 45, 70, and 95°C. Rehydration lasted from 6 h (at 20°C) to 2 h (at 95°C). The initial mass of the dried material subjected to rehydration amounted to ca. 10 g.

Mass determination process was conducted as follows: samples were weighted at least 7 times during the rehydration. Rehydrated samples were separated from the distilled water, dried with the blotting and weighted. The samples were next used to measure dry matter of solid content. Dry matter of solid was determined according to AOAC (2003) standards and was determined 7 times during the rehydration. The mass and dry matter measurements were replicated three times. The WPE 300 scales (RADWAG, Radom) were used for the measurements. The maximum relative error in the determination of the mass and dry matter mass amounted to 0.1%.

The moisture content (dry basis) M was calculated from formula

$$M(\tau) = \frac{m(\tau) - m_{\text{d.m.}}}{m_{\text{d.m.}}} \qquad (1)$$

where: $m(\tau)$ is the mass (g), τ is the time (s), $m_{\text{d.m.}}$ is the mass of dry matter (g).

The moisture ratio was calculated from formula

$$\text{MR}(\tau) = \frac{M(\tau) - M_0}{M_e - M_0} \qquad (2)$$

where: M_0 is the initial moisture content (dry basis), M_e is the moisture content at saturation (equilibrium moisture content, dry basis).

The Weibull model was used to describe the rehydration kinetics

$$\text{MR}(\tau) = 1 - \exp\left[-\left(\frac{\tau}{\alpha}\right)^{\beta}\right] \qquad (3)$$

where: α is the scale parameter (s), β is the dimensionless shape parameter.

The goodness of fit of the model to the experimental data was evaluated with the determination coefficient (R^2), mean bias error (MBE), root mean square error (RMSE), reduced chi-square (χ^2), and t-statistic method. The higher the R^2 value and lower the values of MBE, RMSE, χ^2, and t-stat, the better the goodness of the fit.

The effect of characteristic particle dimension L (mm), drying air velocity v (m/s), drying air temperature t_d (°C), and rehydration temperature t_r (°C) on parameters α and β was also investigated by multiple regression analysis. The parameters were determined by investigating the four-variable polynomial function. The regression analyses and ANOVA ($p<0.05$) were done using the STATISTICA routine.

RESULTS AND DISCUSSION

Plot for variation in mass, dry matter of solid, and moisture content with time during rehydration are shown in Figure 1. It can be seen from Figure 1a and Figure 1c that moisture uptake increases with increasing rehydration time, and the rate is faster in initial period of rehydration and decreased up to the saturation level. This initial period of high water uptake can be attributed to the capillaries and cavities near the surface filling up rapidly (Cunningham et al. 2008). It can be observed from Figure 1b that solute loss increases with increasing rehydration time, and the rate is faster in the initial period of rehydration and decreased up to the saturation level. The explanation of such a course of variation in dry matter of solid with time can be the following. There is an initial steep decrease in solid content because of a high rate of mass transfer. As the solute concentration equilibrated with the environment, the rate of change of solid dry matter is substantially reduced (Sopade et al., 2007).

The results of statistical analyses undertaken on the Weibull model are given in Table 1. As can be seen from the statistical analysis results, generally high determination coefficient R^2 (0.8319-0.9957) and low values of MBE (-0.0044-0.0108), RMSE (0.0189-0.1248), χ^2 (0.0004-0.0180), and t-stat (0.0149-0.2875) were found for Weibull model. Therefore, it can be accepted that considered model may be assumed to represent the rehydration behaviour of dried apples.

Further regressions were undertaken to account for the effect of L, v, t_d, and t_r on the Weibull model parameters α and β. It turned out from the ANOVA ($p<0.05$) that polynomial function takes the following form (empirical formulas)

$$\alpha = 4.454204 - 0.160544\,L^2 - 0.003603\,t_d t_r + 0.001113\,L t_d t_r \quad (4)$$

$$\beta = 0.591161 - 0.000117\,t_r^2 + 0.000209\,t_d t_r v + 0.000068\,L t_d t_r - 0.004333\,L t_r v \quad (5)$$

where: L in mm, v in m/s, t_d in °C, t_r in°C, and L depends not only on the particle dimension but the particle shape as well and according to Pabis et al. (1998): $L=s$ for slice of thickness 2s and $L^{-2}=3s^{-2}$ for cube thickness 2s.
As can be seen the values of scale parameter α and shape parameter β depend on characteristic particle dimension, drying air temperature, and rehydration temperature. The values of β depend moreover on drying air velocity (Eq. (5)).

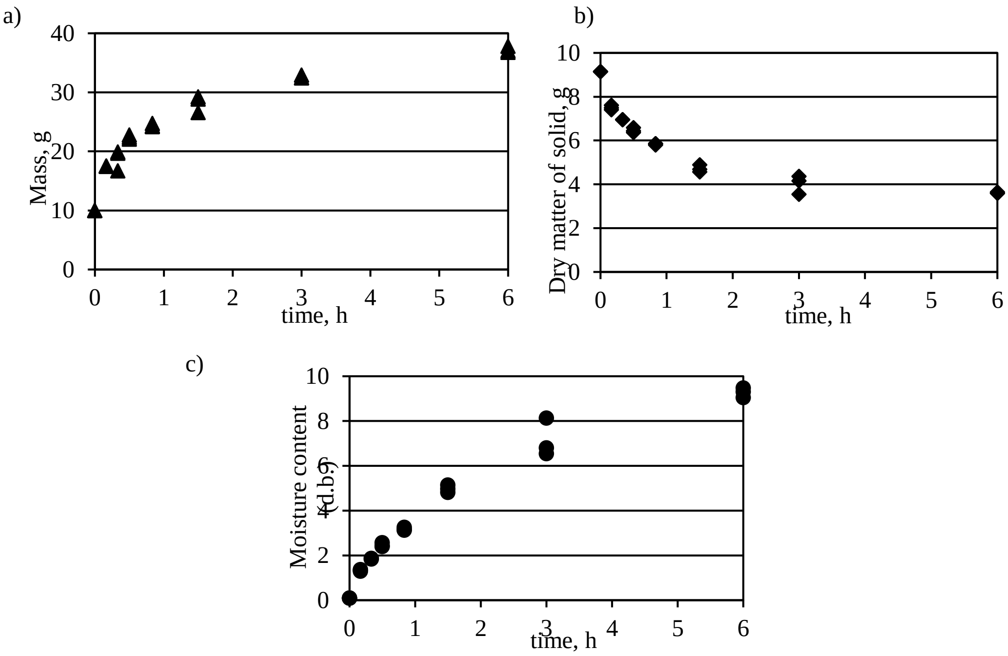

Figure. 1. Variation in a) mass, b) dry matter of solid, c) moisture content with time during rehydration of dried in natural convection at 60°C apple cubes (10 mm) immersed in distilled water at 20°C

Table 1. Results of statistical analyses on the modelling of dried apple rehydration using Weibull model

No.	Conditions of drying and rehydration	Model parameters		R^2	MBE	RMSE	χ^2	t-stat
		α	β					
1	t_r=20ºC, t_d=60ºC, 6 m/s, 10 mm cube	2.6556	0.7334	0.9775	-0.0012	0.0412	0.0019	0.0294
2	t_r=45ºC, t_d=60ºC, 6 m/s, 10 mm cube	2.2407	0.9783	0.9745	0.0038	0.0472	0.0026	0.0813
3	t_r=70ºC, t_d=60ºC, 6 m/s, 10 mm cube	1.9664	0.8266	0.9935	0.0034	0.0247	0.0007	0.1379
4	t_r=95ºC, t_d=60ºC, 6 m/s, 10 mm cube	0.7813	0.6433	0.9793	0.0034	0.0390	0.0017	0.0885
5	t_r=20ºC, t_d=60ºC, 0.01 m/s, 10 mm cube	2.5198	0.8522	0.9899	0.0028	0.0285	0.0009	0.0975
6	t_r=45ºC, t_d=60ºC, 0.01 m/s, 10 mm cube	2.1081	0.9301	0.9949	0.0005	0.0210	0.0005	0.0247
7	t_r=70ºC, t_d=60ºC, 0.01 m/s, 10 mm cube	1.5503	0.7965	0.9930	0.0008	0.0247	0.0007	0.0317
8	t_r=95ºC, t_d=60ºC, 0.01 m/s, 10 mm cube	0.8696	0.6847	0.9809	0.0092	0.0333	0.0013	0.2875
9	t_r=20ºC, t_d=60ºC, 0.01 m/s, 10 mm slice	2.6203	1.0521	0.9506	0.0061	0.0696	0.0053	0.0882
10	t_r=20ºC, t_d=50ºC, 0.5 m/s, 10 mm slice	2.6692	0.7747	0.9887	0.0040	0.0290	0.0009	0.1392
11	t_r=20ºC, t_d=60ºC, 0.5 m/s, 10 mm slice	2.4258	0.7353	0.9951	0.0026	0.0197	0.0004	0.1323
12	t_r=45ºC, t_d=60ºC, 0.5 m/s, 10 mm slice	1.8658	0.8522	0.9691	-0.0008	0.0535	0.0034	0.0149
13	t_r=70ºC, t_d=60ºC, 0.5 m/s, 10 mm slice	1.4147	0.7321	0.9957	0.0017	0.0189	0.0005	0.0921
14	t_r=95ºC, t_d=60ºC, 0.5 m/s, 10 mm slice	0.6999	0.7176	0.9895	0.0032	0.0286	0.0010	0.1111
15	t_r=20ºC, t_d=70ºC, 0.5 m/s, 10 mm cube	3.0272	0.7559	0.9901	0.0032	0.0259	0.0007	0.1244
16	t_r=20ºC, t_d=60ºC, 0.5 m/s, 3 mm slice	1.7715	0.7205	0.9734	-0.0045	0.0474	0.0025	0.0943
17	t_r=20ºC, t_d=60ºC, 2 m/s, 10 mm cube	2.4364	0.7554	0.9934	0.0034	0.0226	0.0006	0.1510
18	t_r=20ºC, t_d=60ºC, 2 m/s, 10 mm slice	2.9630	0.7507	0.9801	0.0017	0.0376	0.0016	0.0440

Figure 2 shows the relationship between the parameters α and β calculated from experiment (their values are shown in Table 1) and predicted by using of Eqs. (4) and (5). The statistical analysis conducted on the Weibull model parameters α and β gives the following results: generally high determination coefficient R^2=0.9165 (for α) and 0.6694 (for β) and low values of MBE=0.0000 and 0.0003, RMSE=0.1849 and 0.0522, χ^2=0.0418 and 0.0035, and t-stat=0.0000 and 0.0064 respectively. Therefore, it can be accepted that considered Eqs. (4) and (5) may be used to represent parameters α and β of the Weibull model.

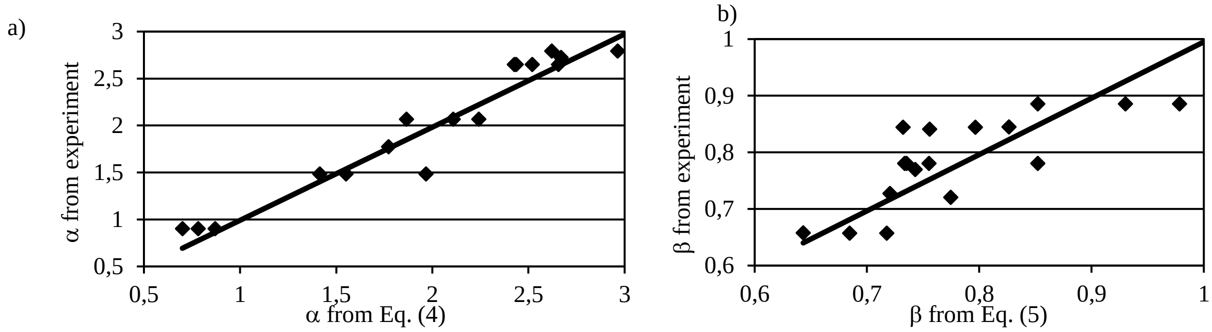

Figure 2. Parameters of the Weibull model calculated from experiment and predicted by using: a) Eq. (4) for scale parameter α, b) Eq. (5) for shape parameter β.

The Eqs. (4) and (5) were then used to estimate the moisture content of rehydrated dried apples at any time during rehydration. Validation of the developed extended Weibull model was made by comparing the computed moisture contents with the measured ones in any particular rehydrating run under certain conditions. The results of statistical analyses undertaken on the extended Weibull model are given in Table 2. They are comparable with the results obtained for the Weibull model (Table 1).

Table 2. Results of statistical analyses on the modelling of dried apple rehydration using extended Weibull model

No.	R^2	MBE	RMSE	χ^2	t-stat
1*)	0.9768	0.0056	0.0430	0.0020	0.1306
2	0.9686	-0.0210	0.0562	0.0036	0.4009
3	0.9913	-0.0051	0.0330	0.0013	0.1555
4	0.9807	0.0280	0.0477	0.0026	0.7265
5	0.9886	0.0020	0.0323	0.0011	0.0618
6	0.9940	-0.0074	0.0230	0.0007	0.3424
7	0.9927	-0.0026	0.0267	0.0009	0.0976
8	0.9814	0.0110	0.0317	0.0012	0.3702
9	0.9486	0.0075	0.0720	0.0057	0.1042
10	0.9870	-0.0012	0.0317	0.0011	0.0389
11	0.9948	0.0227	0.0294	0.0011	1.2214
12	0.9666	0.0206	0.0578	0.0042	0.3805
13	0.9913	0.0207	0.0311	0.0016	0.8958
14	0.9912	0.0377	0.0513	0.0031	1.0801
15	0.9882	-0.0118	0.0391	0.0017	0.3157
16	0.9734	-0.0037	0.0474	0.0025	0.0778
17	0.9942	0.0211	0.0298	0.0010	1.0014
18	0.9797	-0.0047	0.0389	0.0017	0.1210

*)Number of conditions according to Table 1

The values of R^2 are high (0.9130-0.9948) and the values of MBE (-0.0209-0.0377), RMSE (0.0230-0.0719), χ^2 (0.0007-0.0057), and t-stat (0.0389-1.2214) are quite low.

Figure 3 shows the relationship between the moisture ratio calculated experimentally (Eq. 2) and the moisture ratio calculated by using the extended Weibull model (Eqs. (3), (4), and (5)). As shown in Fig. 3 it can be accepted that the extended Weibull model provided a good fit to moisture ratio calculated experimental data for rehydration of dried apples (R^2=0.9776, MBE=0.0081, RMSE=0.0620, χ^2=0.0040, and t-stat=0.3477).

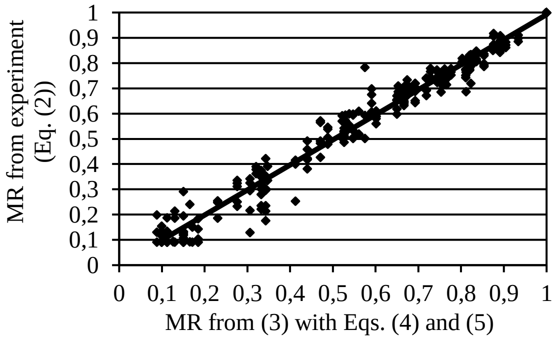

Figure 3. Experimentally determined and predicted from the extended Weibull model moisture content of rehydrated dried apples

This shows the suitability of the extended Weibull model for explaining the kinetics of rehydration of dried apples. Although both Weibull and extended Weibull model are empirical ones, the extended model by taking into account the effect of process conditions (L, v, t_d, t_r) on model parameters can be considered as more general.

CONCLUSIONS

1. Moisture uptake and solute loss increases with increasing rehydration time, and the rate is faster in the initial period of rehydration and decreased up to the saturation level.
2. The Weibull model can be considered as the appropriate for describing the rehydration of dried apple ($R^2 \in$(0.8319-0.9957), MBE$\in$(-0.0044-0.0108), RMSE$\in$(0.0189-0.1248), $\chi^2 \in$(0.0004-0.0180), t-stat$\in$(0.0149-0.2875)).
3. Developed empirical formulas can be considered as appropriate for describing the Weibull model constants α (R^2=0.9165, MBE=0.0000, RMSE=0.1849, χ^2=0.0418, t-stat=0.0000) and β (R^2=0.6694, MBE=0.0003, RMSE= 0.0522, χ^2= 0.0035, t-stat=0.0064)
4. The extended Weibull model can be considered as the appropriate for describing the rehydration of dried apple (R^2=0.9776, MBE=0.0081, RMSE=0.0620, χ^2=0.0040, t-stat=0.3477).
5. The developed extended Weibull model by taking into account the effect of characteristic particle dimension, drying air velocity, drying air temperature, and rehydration temperature on model parameters can be considered as more general one.

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